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Research Article

Stability Analysis and Stabilization of T-S Fuzzy Delta Operator Systems with Time-Varying Delay via an Input-Output Approach

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The stability analysis and stabilization of Takagi-Sugeno (T-S) fuzzy delta operator systems with time-varying delay are investigated via an input-output approach. A model transformation method is employed to approximate the time-varying delay. The original system is transformed into a feedback interconnection form which has a forward subsystem with constant delays and a feedback one with uncertainties. By applying the scaled small gain (SSG) theorem to deal with this new system, and based on a Lyapunov Krasovskii functional (LKF) in delta operator domain, less conservative stability analysis and stabilization conditions are obtained. Numerical examples are provided to illustrate the advantages of the proposed method.

1. Introduction

The T-S fuzzy modeling approach, as a simple and effective tool for nonlinear control systems, has been widely accepted and extensive studied for a few decades [1–8]. In addition, it is well known that time delay is a source of instability or performance degradation [9]. Hence, analysis and synthesis of time-delay systems and other relative studies have attracted much attention during the past years [10–17]. Moreover, high-speed digital processing methods are of increasing importance in modern industrial applications. However, most traditional signal processing and control algorithms are inherently ill-conditioned when data are taken at high sampling rates [18]. The delta operator model can be applied as a useful approach to deal with discrete-time systems under high sampling rates through the analysis methods of continuous-time systems [19–22]. In view of the above considerations, both T-S fuzzy modeling approach and delta operator modeling approach have been extended to tackle the analysis and synthesis of nonlinear systems with time delay [23–25].

Recently, some works on analysis and design of T-S fuzzy systems via delta operator approach were developed [26–28]. However, to the authors' best knowledge, few results on the stability analysis and stabilization for Takagi-Sugeno (T-S) fuzzy delta operator systems with time-varying delay are proposed.

In this paper, an indirect approach, namely, the *input-output* (IO) approach is introduced to deal with the stability analysis and control design of T-S fuzzy delta operator systems with time-varying delay. The main contribution of paper is that the stability analysis and stabilization problems for fuzzy delta operator systems with time-varying delay are investigated by the IO approach. A model approximation method is employed to transform the original system into an equivalent interconnected system, which is comprised of a forward subsystem with constant time delays and a feedback one with delayed uncertainties. The scaled small gain (SSG) method is applied and an LKF in delta domain is constructed to analyze and synthesize this system. Furthermore, a frequency sweeping method [9] is suggested to guarantee the internal stability for the forward subsystem, such that less conservative results are ensured. Finally, some comparisons are made with the existing results and control of a truck-trailer model is also presented to illustrate the effectiveness of our method.

This paper is organized as follows. A model transformation method and the proof of the SSG theorem for T-S fuzzy delta operator systems with time-varying delay are presented in Section 2. In Section 3, the stability analysis and stabilization results are provided. The simulation studies are given in Section 4 to illustrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

Notations. The notations used throughout this paper are standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and $n \times m$ real matrices, respectively. $\mathbf{G}_1 \circ \mathbf{G}_2$ represents the series connection of mapping $\mathbf{G}_1$ and $\mathbf{G}_2$. The notation $P > 0$ (≥ 0) means that the matrix P is positive (semi) definite, I_n denotes an identity matrix with dimension n, and $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix. The symbol "$*$" in a matrix stands for the transposed elements in the symmetric positions.

2. Model Description and Problem Formulation

In the following, we consider a fuzzy delta operator system with time-varying delay, which can be described by the following T-S fuzzy model.

Plant Rule i. **IF** $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and ... and $\theta_p(t)$ is M_{ip}, **THEN**

$$\begin{gathered} \partial x(t) = A_i x(t) + A_{di} x(t - nT) + B_i u(t), \\ x(t) = \phi(t), \quad t \in [-h_2, 0], \ i = 1, 2, \ldots, r, \end{gathered} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state variable; $u(t) \in \mathbb{R}^m$ is control input; n is a time-varying integer; T is the sampling period; the bounded time-varying delay nT satisfies $0 < h_1 \leq nT \leq h_2$; $\phi(t) \in \mathbb{R}^{n_x}$ is the vector-valued initial condition; M_{ij} is the fuzzy set; r is the number of IF-THEN rules; $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]$ are the premise variables which do not depend on the control input; A_i, A_{di}, and B_i are known constant matrices with appropriate dimensions; $\partial x(t)$ is the delta operator of $x(t)$, which is defined by

$$\partial x(t) = \begin{cases} \dfrac{d}{dt} x(t), & T = 0, \\ \dfrac{x(t+T) - x(t)}{T}, & T \neq 0. \end{cases} \tag{2}$$

The overall T-S fuzzy delta operator system with time-varying delay is inferred as follows:

$$\partial x(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \left[A_i x(t) + A_{di} x(t - nT) + B_i u(t) \right], \tag{3}$$

where $\sum_{i=1}^{r} \lambda_i(\theta(t)) = 1$, $\lambda_i(\theta(t)) = \omega_i(\theta(t)) / \sum_{i=1}^{r} \omega_i(\theta(t)) \geq 0$, and $\omega_i(\theta(t)) = \prod_{j=1}^{r} M_{ij}(\theta_j(t))$ with $M_{ij}(\theta_j(t))$ represent the grade of membership of $\theta_j(t)$ in M_{ij}.

The following control law is employed to deal with the problem of stabilization via state feedback, where the controller rule shares the same fuzzy sets with the T-S model.

Controller Rule i. **IF** $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and ... and $\theta_p(t)$ is M_{ip}, **THEN**

$$\begin{aligned} u(t) = {} & K_{1i} x(t) + \frac{1}{2} K_{2i} x(t - h_1) \\ & + \frac{1}{2} K_{3i} x(t - h_2), \quad i = 1, 2, \ldots, r. \end{aligned} \tag{4}$$

The overall T-S fuzzy state feedback control law is inferred as

$$\begin{aligned} u(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \Big[& K_{1i} x(t) + \frac{1}{2} K_{2i} x(t - h_1) \\ & + \frac{1}{2} K_{3i} x(t - h_2) \Big]. \end{aligned} \tag{5}$$

Remark 1. It is noted that the controller given in (5) covers the special cases of the memoryless controller when $K_{2i} = K_{3i} = 0$ and the purely delayed controller when $K_{1i} = 0$, respectively.

Combining system (3) with the control law (5), the resulting closed-loop system can be expressed as follows:

$$\begin{aligned} \partial x(t) = {} & \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\theta(t)) \lambda_j(\theta(t)) \\ & \times \Big[\left(A_i + B_i K_{1j} \right) x(t) + A_{di} x(t - nT) \\ & + \frac{1}{2} B_i K_{2j} x(t - h_1) + \frac{1}{2} B_i K_{3j} x(t - h_2) \Big]. \end{aligned} \tag{6}$$

Before ending this section, we introduce the following lemmas as to be used to prove our main results in the following sections.

Lemma 2 (see [9]). *Consider an interconnected system with two subsystems $\widetilde{\mathcal{S}}_1$ and $\widetilde{\mathcal{S}}_2$:*

$$\begin{aligned} \widetilde{\mathcal{S}}_1 : \ & z(t) = \mathbf{G} \omega(t), \\ \widetilde{\mathcal{S}}_2 : \ & \omega(t) = \Delta z(t), \end{aligned} \tag{7}$$

where the forward subsystem $\widetilde{\mathcal{S}}_1$ is known, the feedback subsystem $\widetilde{\mathcal{S}}_2$ is unknown and time-varying, and assume that $\widetilde{\mathcal{S}}_1$ is internally stable. The closed-loop system formed by $\widetilde{\mathcal{S}}_1$ and $\widetilde{\mathcal{S}}_2$ is asymptotically stable for all $\Delta \in D \triangleq \{\Delta : \|\Delta\|_\infty \leq 1\}$

if there exist matrices $\{T_w, T_z\} \in \mathbb{T}$ *satisfied:*

$$\mathbb{T} \triangleq \Big\{ \{T_w, T_z\} \in \mathbb{R}^{w\times w} \times \mathbb{R}^{z\times z} : T_w, T_z \text{ nonsingular}; \left\| T_w \circ \Delta \circ T_z^{-1} \right\|_\infty \leq 1 \Big\}, \tag{8}$$

such that the following SSG condition holds:

$$\left\| T_z \circ \mathbf{G} \circ T_w^{-1} \right\|_\infty \leq 1. \tag{9}$$

Lemma 3 (see [29]). *For any constant positive semidefinite symmetric matrix* W*, two positive integers* r *and* r_0 *satisfying* $r \geq r_0 \geq 1$*, the following inequality holds:*

$$\left[\sum_{i=r_0}^{r} x(i)\right]^T W \left[\sum_{i=r_0}^{r} x(i)\right] \leq \left(r - r_0 + 1\right) \sum_{i=r_0}^{r} x^T(i) W x(i). \tag{10}$$

Lemma 4 (see [30]). *The property of delta operator: for any time function* $x(t)$ *and* $y(t)$*, it holds that*

$$\partial\left(x(t)\, y(t)\right) = \partial x(t)\, y(t) + x(t)\, \partial y(t) + T \partial x(t)\, \partial y(t), \tag{11}$$

where T *is the sampling period.*

3. Model Transformation

In this paper, the T-S fuzzy delta operator system with time-varying delay is investigated by an IO approach. By this method, the term $x(t - nT)$ is approximated and the error is written into the feedback path. The recent work in [31] proposed a two-term approximation method $(1/2)[x(t - h_1) + x(t - h_2)]$ for $x(t - nT)$, which results in a smaller approximation error bound. Inspired by this method, the approximation error of time-varying delay can be expressed as

$$\begin{aligned}
\omega_t(t) &= x(t - nT) - \frac{1}{2}\left[x\left(t - h_1\right) + x\left(t - h_2\right)\right] \\
&= \frac{T}{2} \sum_{i=-h_2/T}^{-n-1} \partial x(t + iT) - \frac{T}{2} \sum_{i=-n}^{-(h_1/T)-1} \partial x(t + iT) \\
&= \frac{T}{2} \sum_{i=-h_2/T}^{-(h_1/T)-1} k(i)\, \partial x(t + iT),
\end{aligned} \tag{12}$$

where $\partial x(t)$ is defined in (2), and

$$k(i) = \begin{cases} 1, & i < -n, \\ -1, & i \geq -n. \end{cases} \tag{13}$$

3.1. Open-Loop Case. Considering the fuzzy delta operator system (3) and setting $u(t) = 0$, we have

$$\partial x(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \left[A_i x(t) + A_{di} x(t - nT)\right]. \tag{14}$$

Employing the two-term approximation method to pull out the uncertainties of time-varying delay, the open-loop system can be written as an interconnected system with a forward subsystem and a feedback one, which is described by

$$\begin{aligned}
\mathcal{S}_1 &: \left[\begin{array}{c} \partial x(t) \\ \hline z(t) \end{array}\right] = \sum_{i=1}^{r} \lambda_i(\theta(t)) \left[\begin{array}{c|c} \Theta_1 & \dfrac{h_{12}}{2} A_{di} X^{-1} \\ \hline X\Theta_1 & X \dfrac{h_{12}}{2} A_{di} X^{-1} \end{array}\right] \times \left[\begin{array}{c} \zeta(t) \\ \hline \omega(t) \end{array}\right], \\
\mathcal{S}_2 &: \omega(t) = X \Delta X^{-1} z(t),
\end{aligned} \tag{15}$$

where $\Theta_1 = [A_i \ \ (1/2)A_{di} \ \ (1/2)A_{di}]$, $\zeta(t) = \mathrm{col}\{x(t) \ \ x(t - h_1) \ \ x(t - h_2)\}$, $h_{12} = h_2 - h_1$, $\omega(t) = (2/h_{12}) X \omega_t(t)$, the scaling matrix $\{X, X\} \in \mathbb{T}$ has the appropriate dimensions, and the operator Δ is the maping $z(t) \to \omega(t)$.

For convenience, we denote $\omega(t) = X\tilde{\omega}(t)$ and $z(t) = X\tilde{z}(t)$. The system (15) can be rewritten as

$$\begin{aligned}
\mathcal{S}_3 &: \left[\begin{array}{c} \partial x(t) \\ \hline \tilde{z}(t) \end{array}\right] = \sum_{i=1}^{r} \lambda_i(\theta(t)) \left[\begin{array}{c|c} \Theta_1 & \dfrac{h_{12}}{2} A_{di} \\ \hline \Theta_1 & \dfrac{h_{12}}{2} A_{di} \end{array}\right] \left[\begin{array}{c} \zeta(t) \\ \hline \tilde{\omega}(t) \end{array}\right], \\
\mathcal{S}_4 &: \tilde{\omega}(t) = \Delta \tilde{z}(t).
\end{aligned} \tag{16}$$

Now, the uncertainties of the time-varying delay have been pulled out from the system (14). Furthermore, the system has been transformed into the interconnection by the forward subsystem and the feedback subsystem. The following result shows that this reformulated system satisfies the following SSG condition.

Lemma 5. *The operator* $\Delta : z(t) \to \omega(t)$ *in system* (15) *satisfies the SSG theorem if there exists the nosingular matrix* $\{X, X\} \in \mathbb{T}$*, such that*

$$\left\| X \Delta X^{-1} \right\| \leq 1. \tag{17}$$

Proof. Following the notations in (12), under the zero initial condition, we have the following inequalities by using the discrete Jensen inequality in Lemma 3:

$$\begin{aligned}
&\sum_{t=0}^{\infty} \omega^T(t)\,\omega(t) \\
&= \left(\frac{T}{h_{12}}\right)^2 \sum_{t=0}^{\infty} \left[\sum_{i=-h_2/T}^{-(h_1/T)-1} k(i)\,\partial x(t+iT)\right]^T \\
&\quad \times X^T X \left[\sum_{i=-h_2/T}^{-(h_1/T)-1} k(i)\,\partial x(t+iT)\right] \\
&\le \frac{T}{h_{12}} \sum_{i=-h_2/T}^{-(h_1/T)-1} \sum_{t=0}^{\infty} \left[\partial x^T(t+iT)\,X^T X \partial x(t+iT)\right] \\
&\le \frac{T}{h_{12}} \sum_{i=-h_2/T}^{-(h_1/T)-1} \sum_{t=0}^{\infty} \left[\partial x^T(t)\,X^T X \partial x(t)\right] \\
&= \sum_{t=0}^{\infty} z^T(t)\,z(t),
\end{aligned} \tag{18}$$

which implies that $\|X\Delta X^{-1}\| \le 1$. The proof is completed. □

3.2. Closed-Loop Case. Employing the two-term approximation method to pull out the uncertainties of time-varying delay, the closed-loop system (6) can also be written as an interconnected system with a forward subsystem and a feedback one, which is described by

$$\begin{aligned}
&\mathcal{S}_5 : \left[\frac{\partial x(t)}{z(t)}\right] \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t))\,\lambda_j(\theta(t)) \left[\begin{array}{c|c} \Theta_2 & \frac{h_{12}}{2} A_{di} X^{-1} \\ \hline X\Theta_2 & X\frac{h_{12}}{2} A_{di} X^{-1} \end{array}\right] \\
&\quad \times \left[\frac{\zeta(t)}{\widetilde{\omega}(t)}\right], \\
&\mathcal{S}_6 : \omega(t) = X\Delta X^{-1} z(t),
\end{aligned} \tag{19}$$

where $\Theta_2 = [(A_i + B_i K_{1j}) \ \ (1/2)(A_{di} + B_i K_{2j}) \ \ (1/2)(A_{di} + B_i K_{3j})]$, and $\zeta(t)$, $\omega(t)$, and $z(t)$ are defined as the same as the open-loop case.

For convenience, we denote $\omega(t) = X\widetilde{\omega}(t)$ and $z(t) = X\widetilde{z}(t)$. The system (19) can be rewritten as

$$\begin{aligned}
&\mathcal{S}_7 : \left[\frac{\partial x(t)}{\widetilde{z}(t)}\right] \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t))\,\lambda_j(\theta(t)) \left[\begin{array}{c|c} \Theta_2 & \frac{h_{12}}{2} A_{di} \\ \hline \Theta_2 & \frac{h_{12}}{2} A_{di} \end{array}\right] \left[\frac{\zeta(t)}{\widetilde{\omega}(t)}\right], \\
&\mathcal{S}_8 : \widetilde{\omega}(t) = \Delta \widetilde{z}(t).
\end{aligned} \tag{20}$$

Remark 6. The definitions of $\omega(t)$ and $z(t)$ for the closed-loop system are the same as the open-loop system, so it is easy to see that the closed-loop system (19) also satisfies the SSG condition.

Now the reformulated systems have been shown to satisfy the SSG condition in both the open-loop and closed-loop cases. Then the systems in (15) and (19) are asymptotically stable if both the forward subsystems are internally stable. Indeed, a frequency sweeping method is often used to check this condition [9].

Lemma 7 (see [9]). *Consider the following system:*

$$\begin{aligned}
&\widetilde{\mathcal{S}}_1 : z(t) = \mathbf{G}\omega(t), \\
&\widetilde{\mathcal{S}}_2 : \omega(t) = \Delta z(t).
\end{aligned} \tag{21}$$

The aforementioned system is internally asymptotically stable if there exist a scalar $\varepsilon > 0$ and a Lyapunov Krasovskii functional $V(t)$ satisfying

$$V(t) > \varepsilon \|x(t)\|^2, \tag{22}$$

such that the functional

$$w(t) = \dot{V}(t) + z^T(t)\,z(t) - \omega^T(t)\,\omega(t) \tag{23}$$

satisfies

$$w(t) \le -\varepsilon\|x(t)\|^2 - \varepsilon\|\omega(t)\|^2. \tag{24}$$

4. Stability Analysis

The previous section presents a model transformation for the original system (3). The open-loop system has been converted into an interconnected system in (15), and the closed-loop system has been converted into (19). In this section, we investigate the asymptotic stability of the system in (15). First, we present the following result for T-S fuzzy delta system with time-varying delay.

Theorem 8. *Consider T-S fuzzy delta operator system in* (14). *Then given scalars $h_2 > h_1 > 0$ and the sampling period $T > 0$,* *the fuzzy delta operator system* (14) *with time-varying delay is asymptotically stable if there exist positive definite symmetric matrices U, P, R_1, R_2, Q_1, Q_2, and Z, such that the following LMIs hold for $i = 1, 2, \ldots, r$:*

$$\Phi_i = \begin{bmatrix} \Phi_i(1,1) & PA_i & \frac{1}{2}PA_{di} & \frac{1}{2}PA_{di} & \frac{h_{12}}{2}PA_{di} \\ * & \Phi_i(2,2) & \frac{1}{2}PA_{di} + \frac{R_1}{h_1} & \frac{1}{2}PA_{di} + \frac{R_2}{h_2} & \frac{h_{12}}{2}PA_{di} \\ * & * & -Q_1 - \frac{1}{4}Z - \frac{R_1}{h_1} & -\frac{1}{4}Z & -\frac{h_{12}}{4}Z \\ * & * & * & -Q_2 - \frac{1}{4}Z - \frac{R_2}{h_2} & -\frac{h_{12}}{4}Z \\ * & * & * & * & -\frac{h_{12}^2}{4}Z - U \end{bmatrix} < 0, \quad (25)$$

where

$$\Phi_i(1,1) = TP + h_1 R_1 + h_2 R_2 - 2P + U,$$
$$\Phi_i(2,2) = PA_i + A_i^T P + Z + \frac{h_{12}}{T}Z + Q_1 + Q_2 - \frac{R_1}{h_1} - \frac{R_2}{h_2}. \quad (26)$$

Proof. Firstly, choosing a Lyapunov-Krasovskii functional candidate in delta domain,

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (27)$$

where

$$\begin{aligned} V_1(t) &= x^T(t)Px(t), \\ V_2(t) &= T\sum_{i=h_1/T}^{h_2/T}\sum_{j=1}^{i} x^T(t-jT)Zx(t-jT), \\ V_3(t) &= T\sum_{i=1}^{h_1/T} x^T(t-iT)Q_1x(t-iT) \\ &\quad + T\sum_{i=1}^{h_2/T} x^T(t-iT)Q_2x(t-iT), \\ V_4(t) &= \sum_{i=1}^{h_1/T}\sum_{j=1}^{i} e^T(t-jT)R_1e(t-jT) \\ &\quad + \sum_{i=1}^{h_2/T}\sum_{j=1}^{i} e^T(t-jT)R_2e(t-jT), \end{aligned} \quad (28)$$

and T is the sampling period, $e(j) = x(j) - x(j+T)$, so that $\partial x(j) = -e(j)/T$ and $e(t-jT) = x(t-jT) - x(t-(j-1)T)$.

Taking the delta operator manipulations of $V_1(t)$ along the trajectory of systems $\mathcal{S}_1$ and $\mathcal{S}_2$, and using Lemma 4, it can be obtained that

$$\begin{aligned} \partial V_1(t) &= \partial^T(x(t))Px(t) + x^T(t)P\partial(x(t)) \\ &\quad + T\cdot\partial^T(x(t))P\partial(x(t)) \\ &= \begin{bmatrix} \partial x(t) \\ x(t) \\ x(t-h_1) \\ x(t-h_2) \\ \tilde{\omega}(t) \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} TP & 0 & 0 & 0 & 0 \\ * & PA_i + A_i^TP & \frac{1}{2}PA_{di} & \frac{1}{2}PA_{di} & \frac{h_{12}}{2}PA_{di} \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} \partial x(t) \\ x(t) \\ x(t-h_1) \\ x(t-h_2) \\ \tilde{\omega}(t) \end{bmatrix}. \end{aligned} \quad (29)$$

Taking the delta operator manipulation of $V_2(t)$, we have

$$\begin{aligned} \partial V_2(t) &= T\cdot\frac{1}{T}\sum_{i=h_1/T}^{h_2/T}\sum_{j=1}^{i} x^T(t-(j-1)T) \\ &\qquad \times Zx(t-(j-1)T) \\ &\quad - T\cdot\frac{1}{T}\sum_{i=h_1/T}^{h_2/T}\sum_{j=1}^{i} x^T(t-iT)Zx(t-iT) \\ &= \sum_{i=h_1/T}^{h_2/T} x^T(t)Zx(t) \\ &\quad - \sum_{i=h_1/T}^{h_2/T} x^T(t-iT)Zx(t-iT) \\ &\le \left(\frac{h_{12}}{T}+1\right)x^T(t)Zx(t) \\ &\quad - x^T(t-nT)Zx(t-nT). \end{aligned} \quad (30)$$

Substituting (12) into (30), we have

$$\partial V_2(t) = \begin{bmatrix} x(t) \\ x(t-h_1) \\ x(t-h_2) \\ \widetilde{\omega}(t) \end{bmatrix}^T \times \begin{bmatrix} Z + \frac{h_{12}}{T} Z & 0 & 0 & 0 \\ * & -\frac{1}{4} Z & -\frac{1}{4} Z & -\frac{h_{12}}{4} Z \\ * & * & -\frac{1}{4} Z & -\frac{h_{12}}{4} Z \\ * & * & * & -\frac{h_{12}^2}{4} Z \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-h_1) \\ x(t-h_2) \\ \widetilde{\omega}(t) \end{bmatrix}. \tag{31}$$

Taking the delta operator manipulation of $V_3(t)$, we have

$$\begin{aligned} \partial V_3(t) = \frac{1}{T} \cdot T \Bigg[& \sum_{i=1}^{h_1/T} x^T(t-(i-1)T) \times Q_1 x(t-(i-1)T) \\ & + \sum_{i=1}^{h_2/T} x^T(t-(i-1)T) \times Q_2 x(t-(i-1)T) \\ & - \sum_{i=1}^{h_1/T} x^T(t-iT) Q_1 x(t-iT) \\ & - \sum_{i=1}^{h_2/T} x^T(t-iT) Q_2 x(t-iT) \Bigg] \\ = {} & x^T(t)(Q_1+Q_2) x(t) \\ & - x^T(t-h_1) Q_1 x(t-h_1) \\ & - x^T(t-h_2) Q_2 x(t-h_2). \end{aligned} \tag{32}$$

Taking the delta operator manipulation of $V_4(t)$ and using Lemma 3, we have

$$\begin{aligned} \partial V_4(t) = \frac{1}{T} \Bigg[& \sum_{i=1}^{h_1/T} \sum_{j=1}^{i} e^T(t-(j-1)T) \times R_1 e(t-(j-1)T) \\ & + \sum_{i=1}^{h_2/T} \sum_{j=1}^{i} e^T(t-(j-1)T) \times R_2 e(t-(j-1)T) \\ & - \sum_{i=1}^{h_1/T} \sum_{j=1}^{i} e^T(t-jT) R_1 e(t-jT) \\ & - \sum_{i=1}^{h_2/T} \sum_{j=1}^{i} e^T(t-jT) R_2 e(t-jT) \Bigg] \\ \leq {} & \frac{h_1}{T^2} e^T(t) R_1 e(t) \\ & - \frac{1}{h_1} \left(\sum_{i=1}^{h_1/T} e(t-iT) \right)^T R_1 \left(\sum_{i=1}^{h_1/T} e(t-iT) \right) \\ & + \frac{h_2}{T^2} e^T(t) R_2 e(t) \\ & - \frac{1}{h_2} \left(\sum_{i=1}^{h_2/T} e(t-iT) \right)^T R_2 \left(\sum_{i=1}^{h_2/T} e(t-iT) \right) \\ = {} & h_1 \partial x^T(t) R_1 \partial x(t) - \frac{1}{h_1} (x(t-h_1) - x(t))^T \\ & \times R_1 (x(t-h_1) - x(t)) + h_2 \partial x^T(t) R_2 \partial x(t) \\ & - \frac{1}{h_2} (x(t-h_2) - x(t))^T R_2 (x(t-h_2) - x(t)). \end{aligned} \tag{33}$$

For the positive definite symmetric matrix P, we have the following equation from (16):

$$\begin{aligned} 0 = - \sum_{i=1}^{r} \lambda_i(\theta(t)) 2\partial^T(x(t)) P & \times \Big[\partial x(t) - A_i x(t) - \frac{1}{2} A_{di} (x(t-h_1) + x(t-h_2)) \\ & - \frac{h_{12}}{2} A_{di} \widetilde{\omega}(t) \Big]. \end{aligned} \tag{34}$$

Substituting (34) into $\partial V(t)$, we have

$$\partial V(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \xi^T \Sigma_{1i} \xi, \tag{35}$$

where

$$\Sigma_{1i} = \begin{bmatrix} \Sigma_{1i}(1,1) & PA_i & \frac{1}{2}PA_{di} & \frac{1}{2}PA_{di} & \frac{h_{12}}{2}PA_{di} \\ * & \Sigma_{1i}(2,2) & \frac{1}{2}PA_{di}+\frac{R_1}{h_1} & \frac{1}{2}PA_{di}+\frac{R_2}{h_2} & \frac{h_{12}}{2}PA_{di} \\ * & * & -Q_1-\frac{1}{4}Z-\frac{R_1}{h_1} & -\frac{1}{4}Z & -\frac{h_{12}}{4}Z \\ * & * & * & -Q_2-\frac{1}{4}Z-\frac{R_2}{h_2} & -\frac{h_{12}}{4}Z \\ * & * & * & * & -\frac{h_{12}^2}{4}Z \end{bmatrix},$$

$$\Sigma_{1i}(1,1) = TP + h_1R_1 + h_2R_2 - 2P,$$

$$\Sigma_{1i}(2,2) = PA_i + A_i^TP + Z + \frac{h_{12}}{T}Z + Q_1 + Q_2 - \frac{R_1}{h_1} - \frac{R_2}{h_2},$$

$$\xi^T = \left[\partial^T(x(t))\, x^T(t)\, x^T(t-h_1)\, x^T(t-h_2)\, \tilde{\omega}^T(t)\right]. \tag{36}$$

Therefore if $\partial V(t) < 0$, there always exists a sufficiently small scalar ε, for $x(t) \neq 0$, such that $\partial V(t) \leq -\varepsilon\|x(t)\|^2$, which indicates that the systems $\mathcal{S}_1$ and $\mathcal{S}_2$ under $\omega(t) = 0$ are asymptotically stable.

Next, to consider the condition $\omega(t) \neq 0$, we denote $U = X^TX > 0$ and it can be expanded in Lemma 7 as

$$\begin{aligned} \mathscr{W} &\triangleq \partial V(t) + z^T(t)z(t) - \omega^T(t)\omega(t) \\ &= \sum_{i=1}^{r}\lambda_i(\theta(t))\xi^T\Sigma_{1i}\xi + \tilde{z}^T(t)X^TX\tilde{z}(t) \\ &\quad - \tilde{\omega}^T(t)X^TX\tilde{\omega}(t) \\ &= \sum_{i=1}^{r}\lambda_i(\theta(t))\xi^T\Sigma_{2i}\xi, \end{aligned} \tag{37}$$

where $\Sigma_{2i} = \Phi_i$. The proof is completed. □

To compare the results obtained by IO approach, we give the following corollary, which is obtained by a direct LKF-based method.

Corollary 9. *Consider T-S fuzzy delta operator system in* (14). *Then given scalars $h_2 > h_1 > 0$ and the sampling period $T > 0$, the fuzzy delta operator system* (14) *with time-varying delay is asymptotically stable if there exist positive definite symmetric matrices P, R_1, R_2, Q_1, Q_2, and Z, matrices $N = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$, $M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$, $S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$, $\widetilde{X} = \begin{bmatrix} \widetilde{X}_{11} & \widetilde{X}_{12} \\ \widetilde{X}_{12}^T & \widetilde{X}_{22} \end{bmatrix}$, and $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}$, such that the following LMIs* (38)-(39) *hold:*

$$\Psi_1 = \begin{bmatrix} -\widetilde{X}_{11} & -\widetilde{X}_{12} & N_1 \\ * & -\widetilde{X}_{22} & N_2 \\ * & * & -\frac{R_2}{T} \end{bmatrix} < 0,$$

$$\Psi_2 = \begin{bmatrix} -\widetilde{X}_{11} & -\widetilde{X}_{12} & M_1 \\ * & -\widetilde{X}_{22} & M_2 \\ * & * & -\frac{R_2}{T} \end{bmatrix} < 0,$$

$$\Psi_3 = \begin{bmatrix} -Y_{11} & -Y_{12} & S_1 \\ * & -Y_{22} & S_2 \\ * & * & -\frac{R_1}{T} \end{bmatrix} < 0, \tag{38}$$

$$\Psi_{4i} = \begin{bmatrix} \Psi_{4i}(1,1) & PA_i & PA_{di} & 0 & 0 \\ * & \Psi_{4i}(2,2) & \Psi_{4i}(2,3) & -S_1 & -M_1 \\ * & * & \Psi_{4i}(3,3) & -S_2 & -M_2 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r, \tag{39}$$

where

$$\begin{aligned} \Psi_{4i}(1,1) &= TP + h_1R_1 + h_2R_2 - 2P, \\ \Psi_{4i}(2,2) &= PA_i + A_i^TP + Z + \frac{h_{12}}{T}Z + Q_1 + Q_2 \\ &\quad + N_1 + N_1^T + S_1 + S_1^T + \frac{h_2}{T}\widetilde{X}_{11} + \frac{h_1}{T}Y_{11}, \\ \Psi_{4i}(2,3) &= PA_{di} - N_1 + N_2^T + M_1 + S_2^T \\ &\quad + \frac{h_2}{T}\widetilde{X}_{12} + \frac{h_1}{T}Y_{12}, \\ \Psi_{4i}(3,3) &= -Z - N_2 - N_2^T + M_2 + M_2^T \\ &\quad + \frac{h_2}{T}\widetilde{X}_{22} + \frac{h_1}{T}Y_{22}. \end{aligned} \tag{40}$$

Proof. To make a fair comparison, we choose the same LKF candidate as in the proof of Theorem 8.

Taking the delta operator manipulations of $V_1(t)$, $V_2(t)$, $V_3(t)$, and $V_4(t)$ along the trajectory of system (14), we have

$$\begin{aligned} \partial V_1(t) &= \partial^T(x(t))Px(t) + x^T(t)P\partial(x(t)) \\ &\quad + T\cdot\partial^T(x(t))P\partial(x(t)) \\ &= \begin{bmatrix} \partial x(t) \\ x(t) \\ x(t-nT) \end{bmatrix}^T \begin{bmatrix} TP & 0 & 0 \\ * & PA_i + A_i^TP & PA_{di} \\ * & * & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} \partial x(t) \\ x(t) \\ x(t-nT) \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}\partial V_2(t) \le{}& \left(\frac{h_{12}}{T}+1\right) x^T(t) Z x(t)\\ &- x^T(t-nT) Z x(t-nT),\\ \partial V_3(t) ={}& x^T(t)(Q_1+Q_2)x(t)\\ &- x^T(t-h_1) Q_1 x(t-h_1)\\ &- x^T(t-h_2) Q_2 x(t-h_2),\\ \partial V_4(t) ={}& h_1 \partial^T(x(t)) R_1 \partial(x(t))\\ &- \frac{1}{T}\sum_{i=1}^{h_1/T} e^T(t-iT) R_1 e(t-iT)\\ &+ h_2 \partial^T(x(t)) R_2 \partial(x(t))\\ &- \frac{1}{T}\sum_{i=1}^{h_2/T} e^T(t-iT) R_2 e(t-iT),\end{aligned} \tag{41}$$

where $e(t-iT)$ is defined in (27).

For a positive definite symmetric matrix P, we have the following equation from (14):

$$\begin{aligned}0 ={}& -\sum_{i=1}^{r} \lambda_i(\theta(t)) 2\partial^T(x(t)) P\\ &\times \left[\partial x(t) - A_i x(t) - A_{di} x(t-nT)\right].\end{aligned} \tag{42}$$

From the definition of $e(t-iT)$, the following equations hold for any matrices N, M, and S with appropriate dimensions:

$$\begin{aligned}0 &= 2\Upsilon^T(t) N\left[x(t) - x(t-nT) + \sum_{i=1}^{n} e^T(t-iT)\right],\\ 0 &= 2\Upsilon^T(t) M\left[x(t-nT) - x(t-h_2) + \sum_{i=n+1}^{h_2/T} e^T(t-iT)\right],\\ 0 &= 2\Upsilon^T(t) S\left[x(t) - x(t-h_1) + \sum_{i=1}^{h_1/T} e^T(t-iT)\right],\end{aligned} \tag{43}$$

where $\Upsilon^T(t) = [x^T(t) x^T(t-nT)]$.

For any appropriate dimensions matrices $\widetilde{X} = \widetilde{X}^T$ and $Y = Y^T$, we have

$$\begin{aligned}0 ={}& \sum_{i=1}^{h_2/T} \Upsilon^T(t)\widetilde{X}\Upsilon(t) - \sum_{i=1}^{h_2/T} \Upsilon^T(t)\widetilde{X}\Upsilon(t)\\ ={}& \frac{h_2}{T}\Upsilon^T(t)\widetilde{X}\Upsilon(t) - \sum_{i=1}^{n} \Upsilon^T(t)\widetilde{X}\Upsilon(t)\\ &- \sum_{i=n+1}^{h_2/T} \Upsilon^T(t)\widetilde{X}\Upsilon(t),\\ 0 ={}& \sum_{i=1}^{h_1/T} \Upsilon^T(t) Y \Upsilon(t) - \sum_{i=1}^{h_1/T} \Upsilon^T(t) Y \Upsilon(t)\\ ={}& \frac{h_1}{T}\Upsilon^T(t) Y \Upsilon(t) - \sum_{i=1}^{h_1/T} \Upsilon^T(t) Y \Upsilon(t).\end{aligned} \tag{44}$$

Substituting (42)–(44) into $\partial V(t)$, we have

$$\begin{aligned}\partial V(t) ={}& \sum_{i=1}^{r} \lambda_i(t) \xi_1^T \Sigma_{3i} \xi_1 + \sum_{i=1}^{n} \Upsilon_1^T(t) \Sigma_4 \Upsilon_1(t)\\ &+ \sum_{i=1}^{h_1/T} \Upsilon_1^T(t) \Sigma_5 \Upsilon_1(t) + \sum_{i=n+1}^{h_2/T} \Upsilon_1^T(t) \Sigma_6 \Upsilon_1(t),\end{aligned} \tag{45}$$

where $\xi_1^T = [\partial x^T(t) \;\; x^T(t) \;\; x^T(t-nT) \;\; x^T(t-h_1) \;\; x^T(t-h_2)]$, $\Upsilon_1^T(t) = [x^T(t) \;\; x^T(t-nT) \;\; e^T(t-iT)]$, $\Sigma_{3i} = \Psi_{4i}$, $\Sigma_4 = \Psi_1$, $\Sigma_5 = \Psi_3$, and $\Sigma_6 = \Psi_2$. Since $\Sigma_{3i} < 0$, $\Sigma_4 < 0$, $\Sigma_5 < 0$, and $\Sigma_6 < 0$ hold, then $\partial V(t) < 0$. The proof is completed. □

5. Stabilization

The previous section presents the criterion for asymptotic stability of fuzzy delta operator open-loop system. In this section, we are interested in designing a controller in (5). By employing the same LKF and applying IO method, the following criteria can be obtained.

Theorem 10. *Consider T-S fuzzy delta operator system* (3) *with the controller in* (5). *Then given scalars $h_2 > h_1 > 0$ and the sampling period $T > 0$, the fuzzy delta operator system with time-varying delay is asymptotically stable if there exist positive definite symmetric matrices G, $\overline{U}$, $\overline{R}_1$, $\overline{R}_2$, $\overline{Q}_1$, $\overline{Q}_2$, and $\overline{Z}$ and matrices $\widetilde{K}_{1i}$, $\widetilde{K}_{2i}$, and $\widetilde{K}_{3i}$, such that the following LMIs hold:*

$$\begin{aligned}&\Phi_{ii} < 0, \quad (1 \le i \le r),\\ &\Phi_{ij} + \Phi_{ji} < 0, \quad (1 \le i < j \le r),\end{aligned} \tag{46}$$

where

$$\Phi_{ij} = \begin{bmatrix} \Phi_{ij}(1,1) & \Phi_{ij}(1,2) & \Phi_{ij}(1,3) & \Phi_{ij}(1,4) & \frac{h_{12}}{2}A_{di}G \\ * & \Phi_{ij}(2,2) & \Phi_{ij}(2,3) & \Phi_{ij}(2,4) & \frac{h_{12}}{2}A_{di}G \\ * & * & \Phi_{ij}(3,3) & -\frac{1}{4}\overline{Z} & -\frac{h_{12}}{4}\overline{Z} \\ * & * & * & \Phi_{ij}(4,4) & -\frac{h_{12}}{4}\overline{Z} \\ * & * & * & * & -\frac{h_{12}^2}{4}\overline{Z}-\overline{U} \end{bmatrix},$$

$$\begin{aligned} \Phi_{ij}(1,1) &= TG + h_1\overline{R}_1 + h_2\overline{R}_2 - 2G + \overline{U}, \\ \Phi_{ij}(1,2) &= A_iG + B_i\widetilde{K}_{1j}, \\ \Phi_{ij}(1,3) &= \frac{1}{2}\left(A_{di}G + B_i\widetilde{K}_{2j}\right), \\ \Phi_{ij}(1,4) &= \frac{1}{2}\left(A_{di}G + B_i\widetilde{K}_{3j}\right), \\ \Phi_{ij}(2,2) &= A_iG + B_i\widetilde{K}_{1j} + GA_i^T + \widetilde{K}_{1j}^TB_i^T \\ &\quad + \overline{Z} + \frac{h_{12}}{T}\overline{Z} + \overline{Q}_1 + \overline{Q}_2 - \frac{\overline{R}_1}{h_1} - \frac{\overline{R}_2}{h_2}, \\ \Phi_{ij}(2,3) &= \frac{1}{2}\left(A_{di}G + B_i\widetilde{K}_{2j}\right) + \frac{\overline{R}_1}{h_1}, \\ \Phi_{ij}(2,4) &= \frac{1}{2}\left(A_{di}G + B_i\widetilde{K}_{3j}\right) + \frac{\overline{R}_2}{h_2}, \\ \Phi_{ij}(3,3) &= -\overline{Q}_1 - \frac{1}{4}\overline{Z} - \frac{\overline{R}_1}{h_1}, \\ \Phi_{ij}(4,4) &= -\overline{Q}_2 - \frac{1}{4}\overline{Z} - \frac{\overline{R}_2}{h_2}. \end{aligned} \tag{47}$$

Moreover, a suitable stabilizing fuzzy state feedback controller can be chosen by

$$\begin{aligned} u(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \Big[& K_{1i}x(t) + \frac{1}{2}K_{2i}x(t-h_1) \\ & + \frac{1}{2}K_{3i}x(t-h_2) \Big], \quad i = 1,2,\ldots,r, \end{aligned} \tag{48}$$

where $K_{1i} = \widetilde{K}_{1i}G^{-1}$, $K_{2i} = \widetilde{K}_{2i}G^{-1}$, $K_{3i} = \widetilde{K}_{3i}G^{-1}$.

Proof. Choosing the same LKF candidate as in the proof of Theorem 8, we have

$$\partial V(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t))\lambda_j(\theta(t))\xi^T\Sigma_{1ij}\xi, \tag{49}$$

where

$$\Sigma_{1ij} = \begin{bmatrix} \Sigma_{1ij}(1,1) & \Sigma_{1ij}(1,2) & \Sigma_{1ij}(1,3) & \Sigma_{1ij}(1,4) & \frac{h_{12}}{2}PA_{di} \\ * & \Sigma_{1ij}(2,2) & \Sigma_{1ij}(2,3) & \Sigma_{1ij}(2,4) & \frac{h_{12}}{2}PA_{di} \\ * & * & \Sigma_{1ij}(3,3) & -\frac{1}{4}Z & -\frac{h_{12}}{4}Z \\ * & * & * & \Sigma_{1ij}(4,4) & -\frac{h_{12}}{4}Z \\ * & * & * & * & -\frac{h_{12}^2}{4}Z \end{bmatrix},$$

$$\begin{aligned} \Sigma_{1ij}(1,1) &= TP + h_1R_1 + h_2R_2 - 2P, \\ \Sigma_{1ij}(1,2) &= P\left(A_i + B_iK_{1j}\right), \\ \Sigma_{1ij}(1,3) &= \frac{1}{2}P\left(A_{di} + B_iK_{2j}\right), \\ \Sigma_{1ij}(1,4) &= \frac{1}{2}P\left(A_{di} + B_iK_{3j}\right), \\ \Sigma_{1ij}(2,2) &= P\left(A_i + B_iK_{1j}\right) + \left(A_i + B_iK_{1j}\right)^TP \\ &\quad + Z + \frac{h_{12}}{T}Z + Q_1 + Q_2 - \frac{R_1}{h_1} - \frac{R_2}{h_2}, \\ \Sigma_{1ij}(2,3) &= \frac{1}{2}P\left(A_{di} + B_iK_{2j}\right) + \frac{R_1}{h_1}, \\ \Sigma_{1ij}(2,4) &= \frac{1}{2}P\left(A_{di} + B_iK_{3j}\right) + \frac{R_2}{h_2}, \\ \Sigma_{1ij}(3,3) &= -Q_1 - \frac{1}{4}Z - \frac{R_1}{h_1}, \\ \Sigma_{1ij}(4,4) &= -Q_2 - \frac{1}{4}Z - \frac{R_2}{h_2}, \end{aligned} \tag{50}$$

and ξ is defined in (35).

Next, by applying Lemma 7, we have

$$\begin{aligned} \mathscr{W} &\triangleq \partial V(t) + z^T(t)z(t) - \omega^T(t)\omega(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\theta(t))\lambda_j(\theta(t))\xi^T\Sigma_{1ij}\xi \\ &\quad + \tilde{z}^T(t)X^TX\tilde{z}(t) - \tilde{\omega}^T(t)X^TX\tilde{\omega}(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\theta(t))\lambda_j(\theta(t))\xi^T\Sigma_{2ij}\xi, \end{aligned} \tag{51}$$

where

$$\Sigma_{2ij} = \begin{bmatrix} \Sigma_{1ij}(1,1)+U & \Sigma_{1ij}(1,2) & \Sigma_{1ij}(1,3) & \Sigma_{1ij}(1,4) & \frac{h_{12}}{2}PA_{di} \\ * & \Sigma_{1ij}(2,2) & \Sigma_{1ij}(2,3) & \Sigma_{1ij}(2,4) & \frac{h_{12}}{2}PA_{di} \\ * & * & \Sigma_{1ij}(3,3) & -\frac{1}{4}Z & -\frac{h_{12}}{4}Z \\ * & * & * & \Sigma_{1ij}(4,4) & -\frac{h_{12}}{4}Z \\ * & * & * & * & -\frac{h_{12}^2}{4}Z-U \end{bmatrix}. \tag{52}$$

It can be clearly shown that

$$\begin{aligned} \mathscr{W} &\triangleq \sum_{i=1}^{r} \lambda_i^2(\theta(t))\,\xi^T\Sigma_{2ii}\xi \\ &\quad + \sum_{i=1}^{r}\sum_{i<j}^{r} \lambda_i(\theta(t))\,\lambda_j(\theta(t))\,\xi^T \times \left(\Sigma_{2ij}+\Sigma_{2ji}\right)\xi. \end{aligned} \tag{53}$$

Premultiplying and postmultiplying Σ_{2ii} by $\mathrm{diag}\{P^{-1}\ P^{-1}\ P^{-1}\ P^{-1}\ P^{-1}\}$, and letting $G = P^{-1}$, $\overline{R}_1 = P^{-1}R_1P^{-1}$, $\overline{R}_2 = P^{-1}R_2P^{-1}$, $\overline{Q}_1 = P^{-1}Q_1P^{-1}$, $\overline{Q}_2 = P^{-1}Q_2P^{-1}$, $\overline{U} = P^{-1}UP^{-1}$, $\widetilde{K}_{1i} = K_{1i}P^{-1}$, $\widetilde{K}_{2i} = K_{2i}P^{-1}$, and $\widetilde{K}_{3i} = K_{3i}P^{-1}$ yield Φ_{ii}. Following a similar line in the previous process to Σ_{2ij} and Σ_{2ji} yields Φ_{ij} and Φ_{ji}.

Since $\Phi_{ii} < 0$ holds for $1 \le i \le r$, and $(\Phi_{ij} + \Phi_{ji}) < 0$ holds for $1 \le i < j \le r$,then we have $\mathscr{W} < 0$. Then by using Lemma 7, the system (19) is internally asymptotically stable. Furthermore, from Lemma 2, the fuzzy delta operator system (3) under the controller (5) is asymptotically stable. Finally, the explicit expression of the state feedback controller is given by $K_{1i} = \widetilde{K}_{1i}G^{-1}$, $K_{2i} = \widetilde{K}_{2i}G^{-1}$, and $K_{3i} = \widetilde{K}_{3i}G^{-1}$. The proof is completed. □

To compare the results obtained by the IO approach, we give the following corollary, which is obtained by a direct LKF-based method.

Corollary 11. *Consider T-S fuzzy delta operator system* (3) *with the controller in* (5). *Then given scalars $h_2 > h_1 > 0$ and the sampling period $T > 0$, the fuzzy delta operator system with time-varying delay is asymptotically stable if there exist positive definite symmetric matrices G, $\overline{R}_1$, $\overline{R}_2$, $\overline{Q}_1$, $\overline{Q}_2$, and $\overline{Z}$ and matrices $\overline{N} = \begin{bmatrix}\overline{N}_1\\ \overline{N}_2\end{bmatrix}$, $\overline{M} = \begin{bmatrix}\overline{M}_1\\ \overline{M}_2\end{bmatrix}$, $\overline{S} = \begin{bmatrix}\overline{S}_1\\ \overline{S}_2\end{bmatrix}$, $\overline{X} = \begin{bmatrix}\overline{X}_{11} & \overline{X}_{12}\\ \overline{X}_{12}^T & \overline{X}_{22}\end{bmatrix}$, $\overline{Y} = \begin{bmatrix}\overline{Y}_{11} & \overline{Y}_{12}\\ \overline{Y}_{12}^T & \overline{Y}_{22}\end{bmatrix}$ and $\widetilde{K}_{1i}$, such that* (38) *and the following LMIs hold:*

$$\begin{gathered} \Psi_{4ii} < 0 \quad (1 \le i \le r), \\ \Psi_{4ij} + \Psi_{4ji} < 0 \quad (1 \le i < j \le r), \end{gathered} \tag{54}$$

where

$$\Psi_{4ij} = \begin{bmatrix} \Psi_{4ij}(1,1) & A_iG+B_i\widetilde{K}_{1j} & A_{di}G & 0 & 0 \\ * & \Psi_{4ij}(2,2) & \Psi_{4ij}(2,3) & -\overline{S}_1 & -\overline{M}_1 \\ * & * & \Psi_{4ij}(3,3) & -\overline{S}_2 & -\overline{M}_2 \\ * & * & * & -\overline{Q}_1 & 0 \\ * & * & * & * & -\overline{Q}_2 \end{bmatrix},$$

$$\begin{aligned} \Psi_{4ij}(1,1) &= TG + h_1\overline{R}_1 + h_2\overline{R}_2 - 2G, \\ \Psi_{4ij}(2,2) &= A_iG + B_i\widetilde{K}_{1j} + GA_i^T + \widetilde{K}_{1j}^TB_i^T + \overline{Z} \\ &\quad + \frac{h_{12}}{T}\overline{Z} + \overline{Q}_1 + \overline{Q}_2 + \overline{N}_1 + \overline{N}_1^T + \overline{S}_1 + \overline{S}_1^T \\ &\quad + \frac{h_2}{T}\overline{X}_{11} + \frac{h_1}{T}\overline{Y}_{11}, \\ \Psi_{4ij}(2,3) &= A_{di}G - \overline{N}_1 + \overline{N}_2^T + \overline{M}_1 + \overline{S}_2^T \\ &\quad + \frac{h_2}{T}\overline{X}_{12} + \frac{h_1}{T}\overline{Y}_{12}, \\ \Psi_{4ij}(3,3) &= -\overline{Z} - \overline{N}_2 - \overline{N}_2^T + \overline{M}_2 + \overline{M}_2^T \\ &\quad + \frac{h_2}{T}\overline{X}_{22} + \frac{h_1}{T}\overline{Y}_{22}. \end{aligned} \tag{55}$$

Moreover, a suitable stabilizing fuzzy state feedback controller is given by

$$u(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\,K_{1i}x(t), \quad i = 1,2,\ldots,r, \tag{56}$$

where $K_{1i} = \widetilde{K}_{1i}G^{-1}$.

Proof. Choosing the same LKF candidate as in the proof of Theorem 8, we have

$$\begin{aligned} \partial V(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t))\,\lambda_j(\theta(t))\,\xi_1^T\Sigma_{3ij}\xi_1 \\ &\quad + \sum_{i=1}^{n} \Upsilon_1^T(t)\,\Sigma_4\Upsilon_1(t) + \sum_{i=1}^{h_1/T} \Upsilon_1^T(t)\,\Sigma_5\Upsilon_1(t) \\ &\quad + \sum_{i=n+1}^{h_2/T} \Upsilon_1^T(t)\,\Sigma_6\Upsilon_1(t), \end{aligned} \tag{57}$$

where

$$\Sigma_{3ij} = \begin{bmatrix} \Sigma_{3ij}(1,1) & P\left(A_i + B_i K_{1j}\right) & PA_{di} & 0 & 0 \\ * & \Sigma_{3ij}(2,2) & \Sigma_{3ij}(2,3) & -S_1 & -M_1 \\ * & * & \Sigma_{3ij}(3,3) & -S_2 & -M_2 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix},$$

$$\Sigma_{3ij}(1,1) = TP + h_1R_1 + h_2R_2 - 2P,$$

$$\Sigma_{3ij}(2,2) = P\left(A_i + B_iK_{1j}\right) + \left(A_i + B_iK_{1j}\right)^T P + Z + \frac{h_{12}}{T}Z + Q_1 + Q_2 + N_1 + N_1^T + S_1 + S_1^T + \frac{h_2}{T}\widetilde{X}_{11} + \frac{h_1}{T}Y_{11},$$

$$\Sigma_{3ij}(2,3) = PA_{di} - N_1 + N_2^T + M_1 + S_2^T + \frac{h_2}{T}\widetilde{X}_{12} + \frac{h_1}{T}Y_{12},$$

$$\Sigma_{3ij}(3,3) = -Z - N_2 - N_2^T + M_2 + M_2^T + \frac{h_2}{T}\widetilde{X}_{22} + \frac{h_1}{T}Y_{22}, \tag{58}$$

and ξ_1^T, Υ_1^T, Σ_4, Σ_5, and Σ_6 are defined in (45).

It can be clearly shown that

$$\begin{aligned} \partial V(t) \triangleq & \sum_{i=1}^{r} \lambda_i^2(\theta(t))\,\xi_1^T \Sigma_{3ii}\xi_1 \\ & + \sum_{i=1}^{r}\sum_{i<j}^{r} \lambda_i(\theta(t))\,\lambda_j(\theta(t))\,\xi_1^T\left(\Sigma_{3ij} + \Sigma_{3ji}\right)\xi_1 \\ & + \sum_{i=1}^{n} \Upsilon_1^T(t)\,\Sigma_4\Upsilon_1(t) + \sum_{i=1}^{h_1/T} \Upsilon_1^T(t)\,\Sigma_5\Upsilon_1(t) \\ & + \sum_{i=n+1}^{h_2/T} \Upsilon_1^T(t)\,\Sigma_6\Upsilon_1(t). \end{aligned} \tag{59}$$

Premultiplying and postmultiplying Σ_{3ii} by $\mathrm{diag}\{P^{-1}\ P^{-1}\ P^{-1}\ P^{-1}\ P^{-1}\}$, premultiplying and postmultiplying $\Sigma_4, \Sigma_5, \Sigma_6$ by $\mathrm{diag}\{P^{-1}\ P^{-1}\ P^{-1}\}$, and letting $G = P^{-1}$, $\overline{R}_1 = P^{-1}R_1P^{-1}$, $\overline{R}_2 = P^{-1}R_2P^{-1}$, $\overline{Q}_1 = P^{-1}Q_1P^{-1}$, $\overline{Q}_2 = P^{-1}Q_2P^{-1}$, $\overline{Z} = P^{-1}ZP^{-1}$, $\overline{N}_1 = P^{-1}N_1P^{-1}$, $\overline{N}_2 = P^{-1}N_2P^{-1}$, $\overline{M}_1 = P^{-1}M_1P^{-1}$, $\overline{M}_2 = P^{-1}M_2P^{-1}$, $\overline{S}_1 = P^{-1}S_1P^{-1}$, $\overline{S}_2 = P^{-1}S_2P^{-1}$, $\overline{X}_{11} = P^{-1}\widetilde{X}_{11}P^{-1}$, $\overline{X}_{12} = P^{-1}\widetilde{X}_{12}P^{-1}$, $\overline{X}_{22} = P^{-1}\widetilde{X}_{22}P^{-1}$, $\overline{Y}_{11} = P^{-1}Y_{11}P^{-1}$, $\overline{Y}_{12} = P^{-1}Y_{12}P^{-1}$, $\overline{Y}_{22} = P^{-1}Y_{22}P^{-1}$, and $\widetilde{K}_{1i} = K_{1i}P^{-1}$ yield Ψ_{4ii}, Ψ_1, Ψ_2, and Ψ_3. Following a similar line of the previous process to Σ_{3ij} and Σ_{3ji} yields Ψ_{4ij} and Ψ_{4ji}.

Since $\Psi_{4ii} < 0$ holds for $1 \le i \le r$, and $(\Psi_{4ij} + \Psi_{4ji}) < 0$ holds for $1 \le i < j \le r$, $\Sigma_4 < 0$, $\Sigma_5 < 0$, and $\Sigma_6 < 0$,then we have $\partial V(t) < 0$. Therefore the fuzzy delta operator system (3) under the controller (59) is asymptotically stable. Finally, the explicit expression of the state feedback controller is given by $K_{1i} = \widetilde{K}_{1i}G^{-1}$. The proof is completed. □

Table 1: Comparisons of maximum allowed delay upper bound h_2 for Example 12 with $h_1 = 0.8$.

Method	h_2 ($T = 0.01$)
Result of Corollary 9	Infeasible
Result of Theorem 8	0.933

6. Simulation Examples

In this section, three examples are provided to demonstrate the effectiveness of the proposed results.

Example 12 (Stability Analysis). Consider a T-S fuzzy delta operator system with time-varying delay in the form of (1) with parameters given by

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}. \tag{60}$$

In this example, for a given delay lower bound $h_1 = 0.8$, we seek for the admissible upper bound h_2, which guarantees the asymptotic stability of the open-loop system. Choosing the sampling period $T = 0.01$, the obtained results are listed in Table 1.

Table 1 shows that the proposed result in Theorem 8 is less conservative than that in Corollary 9, which demonstrates the advantages of our method. Table 2 shows the delay upper bound h_2 under different delay lower bound h_1 and different sampling period T. It is obvious that the delay upper bound h_2 increases gradually as the sampling rate rises, which indicates the advantage of the delta operator fuzzy system at high sampling rate.

Example 13 (Controller Design). To further illustrate the effectiveness of our method for controller design, we consider the following T-S fuzzy delta operator system with time-varying delay:

$$\partial V(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\left[A_i x(t) + A_{di}x(t - nT) + B_i u(t)\right], \tag{61}$$

where A_i, B_i, and A_{di} $(i = 1, 2)$ are given by

$$A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{62}$$

Table 2: Comparisons of maximum allowed delay upper bound h_2 by different h_1 and T for Example 12.

Method	$T = 0.01$			$T = 0.05$			$T = 0.1$		
h_1	0.1	0.4	0.8	0.1	0.4	0.8	0.1	0.4	0.8
Result of Theorem 8	0.732	0.790	0.933	0.685	0.749	0.901	0.626	0.700	0.863

Table 3: Comparisons of maximum allowed delay upper bound h_2 by different h_1 and T for Example 13.

Method	$T = 0.01$			$T = 0.05$			$T = 0.1$		
h_1	0.1	0.4	0.8	0.1	0.4	0.8	0.1	0.4	0.8
Result of Corollary 11	0.185	—	—	0.179	—	—	0.172	—	—
Result of Theorem 10	0.428	0.668	0.951	0.418	0.658	0.941	0.406	0.646	0.929

For different delay lower bounds h_1, the allowed delay upper bounds h_2 are listed in Table 3. It can be seen that the proposed results in Theorem 10 are less conservative than those in Corollary 11.

The fuzzy controller gains for $T = 0.01$, $h_1 = 0.8$, and $h_2 = 0.951$ by Theorem 10 are given as

$$\begin{aligned} K_{11} &= [1.2781 \quad -4.4103], \\ K_{12} &= [0.0812 \quad -2.9757], \\ K_{21} &= [-0.1010 \quad -2.0993], \\ K_{22} &= [-1.0592 \quad -2.5416], \\ K_{31} &= [-0.1064 \quad -1.9295], \\ K_{32} &= [-1.0592 \quad -2.5414]. \end{aligned} \tag{63}$$

Example 14. To illustrate the application of our method, we consider the following truck-trailer system given in [32]:

$$\begin{aligned} \dot{x}_1(t) &= -c\frac{vt_1}{Lt_0}x_1(t) - (1-c)\frac{vt_1}{Lt_0}x_1(t-d(t)) + \frac{vt_1}{lt_0}u(t), \\ \dot{x}_2(t) &= c\frac{vt_1}{Lt_0}x_1(t) + (1-c)\frac{vt_1}{Lt_0}x_1(t-d(t)), \\ \dot{x}_3(t) &= \frac{vt_1}{t_0}\sin x_2(t) + c\frac{vt_1}{2L}x_1(t) \\ &\quad + (1-c)\frac{vt_1}{2L}x_1(t-d(t)), \end{aligned} \tag{64}$$

where $x_1(t)$ is the angular difference between the truck and trailer, $x_2(t)$ is the angle of the trailer, and $x_3(t)$ is the vertical position of rear end of the trailer.

The model parameters are given as $l = 2.8$, $L = 5.5$, $v = -1.0$, $t_1 = 2.0$, and $t_0 = 0.5$, and $c \in [0,1]$ is a retarded coefficient with limits 0 and 1 corresponding to delay-free term and to a full-delay term. The premise variable is chosen as $\theta(t) = x_2(t) + c(vt_1/Lt_0)x_1(t) + (1-c)(vt_1/Lt_0)x_1(t-d(t))$, and the sampling period $T = 0.01$. The following fuzzy rules via delta operator are employed by

Plant Rule 1: IF $\theta(t) =$ is about 0 rad**, THEN**

$$\partial x(t) = A_1x(t) + A_{d1}x(t-nT) + B_1u(t), \tag{65}$$

Plant Rule 2: IF $\theta(t) =$ is about π rad or -π rad**, THEN**

$$\partial x(t) = A_2x(t) + A_{d2}x(t-nT) + B_2u(t). \tag{66}$$

The membership functions for Rule 1 and Rule 2 are given by

$$\lambda = \begin{cases} \lambda_1 = \left(1 - \dfrac{1}{1+\exp(-3(\theta(t)-0.5\pi))}\right) \\ \qquad \times\left(1 - \dfrac{1}{1+\exp(-3(\theta(t)+0.5\pi))}\right), \\ \lambda_2 = 1-\lambda_1, \end{cases} \tag{67}$$

and with

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.509 & 0 & 0 \\ -0.509 & 0 & 0 \\ 0.509 & -4 & 0 \end{bmatrix}, \\ A_{d1} &= \begin{bmatrix} 0.218 & 0 & 0 \\ -0.218 & 0 & 0 \\ 0.218 & 0 & 0 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} -1.4286 \\ 0 \\ 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0.509 & 0 & 0 \\ -0.509 & 0 & 0 \\ 0.810 & -6.366 & 0 \end{bmatrix}, \\ A_{d2} &= \begin{bmatrix} 0.218 & 0 & 0 \\ -0.218 & 0 & 0 \\ 0.347 & 0 & 0 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -1.4286 \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \tag{68}$$

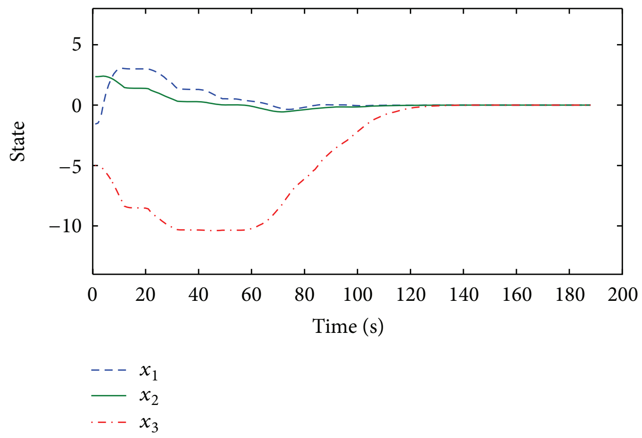

FIGURE 1: State responses for the closed-loop system in Example 14.

Assume the time-varying delay $1 \leq nT \leq 2$, and the initial condition $x(t_0) = [-0.5\pi \;\; 0.75\pi \; -5]^T$. The fuzzy delta operator controller gains by Theorem 10 are given as

$$\begin{aligned} K_{11} &= \begin{bmatrix} 2.2108 & -2.7252 & 0.1445 \end{bmatrix}, \\ K_{12} &= \begin{bmatrix} 2.2620 & -3.0776 & 0.1453 \end{bmatrix}, \\ K_{21} &= \begin{bmatrix} 0.5423 & 0.0608 & -0.0046 \end{bmatrix}, \\ K_{22} &= \begin{bmatrix} 0.5656 & 0.0503 & -0.0038 \end{bmatrix}, \\ K_{31} &= \begin{bmatrix} 0.5467 & 0.0095 & -0.0015 \end{bmatrix}, \\ K_{32} &= \begin{bmatrix} 0.5689 & 0.0079 & -0.0012 \end{bmatrix}. \end{aligned} \tag{69}$$

As shown in Figure 1, the states of the closed-loop system converge to zero under the obtained fuzzy delta operator state-feedback controller, which demonstrates the effectiveness of our method.

7. Conclusion

This paper proposes an input-output method to analysis and synthesis of T-S fuzzy delta operator systems with time-varying delay. The two-term approximation method has been employed to transform the fuzzy delta operator system with time-varying delay into a feedback interconnection form. Based on a Lyapunov-Krasovskii functional in delta operator domain, the SSG method is suggested for the interconnected system. Numerical examples are given to demonstrate the advantages and less-conservatism of the proposed results.

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